Conversion Factors from BG to SI Units

	To convert from	To	Multiply by
Acceleration	ft/s^2	m/s^2	0.3048
Area	ft^2	m^2	9.2903 E − 2
	mi^2	m^2	2.5900 E + 6
	acres	m^2	4.0469 E + 3
Density	$slug/ft^3$	kg/m^3	5.1538 E + 2
	lbm/ft^3	kg/m^3	1.6019 E + 1
Energy	ft-lbf	J	1.3558
	Btu	J	1.0551 E + 3
	cal	J	4.1868
Force	lbf	N	4.4482
	kgf	N	9.8067
Length	ft	m	0.3048
	in	m	2.5400 E − 2
	mi (statute)	m	1.6093 E + 3
	nmi (nautical)	m	1.8520 E + 3
Mass	slug	kg	1.4594 E + 1
	lbm	kg	4.5359 E − 1
Mass flow	slug/s	kg/s	1.4594 E + 1
	lbm/s	kg/s	4.5359 E − 1
Power	ft·lbf/s	W	1.3558
	hp	W	7.4570 E + 2

Conversion Factors from BG to SI Units (Continued)

	To convert from	To	Multiply by
Pressure	lbf/ft^2	Pa	4.7880 E + 1
	lbf/in^2	Pa	6.8948 E + 3
	atm	Pa	1.0133 E + 5
	mm Hg	Pa	1.3332 E + 2
Specific weight	lbf/ft^3	N/m^3	1.5709 E + 2
Specific heat	$ft^2/(s^2 \cdot °R)$	$m^2/(s^2 \cdot K)$	1.6723 E − 1
Surface tension	lbf/ft	N/m	1.4594 E + 1
Temperature	°F	°C	$t_C = \frac{5}{9}(t_F - 32°)$
	°R	K	0.5556
Velocity	ft/s	m/s	0.3048
	mi/h	m/s	4.4704 E − 1
	knot	m/s	5.1444 E − 1
Viscosity	$lbf \cdot s/ft^2$	$N \cdot s/m^2$	4.7880 E + 1
	$g/(cm \cdot s)$	$N \cdot s/m^2$	0.1
Volume	ft^3	m^3	2.8317 E − 2
	L	m^3	0.001
	gal (U.S.)	m^3	3.7854 E − 3
	fluid ounce (U.S.)	m^3	2.9574 E − 5
Volume flow	ft^3/s	m^3/s	2.8317 E − 2
	gal/min	m^3/s	6.3090 E − 5

Quantity	Symbol	Dimen
		$MLT\Theta$
Length	L	L
Area	A	L^2
Volume	$\mathcal{V}$	L^3
Velocity	V	LT^{-1}
Acceleration	dV/dt	LT^{-2}
Speed of sound	a	LT^{-1}
Volume flow	Q	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$
Strain rate	$\dot{\epsilon}$	T^{-1}
Angle	θ	None
Angular velocity	ω, Ω	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$
Kinematic viscosity	ν	L^2T^{-1}
Surface tension	Υ	MT^{-2}
Force	F	MLT^{-2}
Moment, torque	M	ML^2T^{-2}
Power	P	ML^2T^{-3}
Work, energy	W, E	ML^2T^{-2}
Density	ρ	ML^{-3}
Temperature	T	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$
Expansion coefficient	β	Θ^{-1}

$SG = P_{gas}/P_{AIR} = \rho_{liq}/\rho_{water}$, $e = E/m = u + gz + V^2/2$

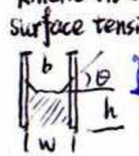
Ideal gas: $p = \rho RT$, $R = R_u/M_{gas}$, $R_u = 8313\,kJ/mol\cdot K$, $R_{air} = 0.287\,kJ/kg\cdot K$

specific heat: $C_v = C_v(T)$ $du = C_v(T)\,dT$ Incompressible: $C_p = C_v = C$, $dh = c\,dT$

Enthalpy: $h = u + P/\rho = u + RT = h(T)$, $dh = C_p(T)\,dT$ heat transfer rate: $q = Q/A = -k\nabla T$

kinetic viscosity: $\nu = \mu/\rho$ Reynolds: $Re = VL/\nu$ shear stress: $\tau = \mu\frac{du}{dy}$

surface tension: $F_{wall} = W_{fluid}$. $F_{wall} = Y\cos\theta \cdot L_{wall}$, $\theta < 90°$ wetting, $> 90°$, nonwet.



$\gamma whb = 2bY\cos\theta$

$\gamma\pi R^2 h = 2\pi RY\cos\theta$

Mach #: $V/a = V/343\,m/s$

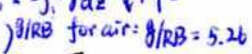
Incompressible: $Ma < 0.3$

stream line: $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \frac{dr}{V}$

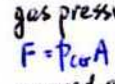
press force/vol: $f_{press} = -\nabla p$ pressure difference: $p_2 - p_1 = -\int_1^2 \gamma\,dz$ $\downarrow + \uparrow -$

gas pressure: $p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$, $p = p_a\left(1 - \frac{Bz}{T_0}\right)^{g/RB}$ for air: $g/RB = 5.26$

$F = P_{cg}A$ on CP, $y_{cp} = -\gamma\sin\theta\frac{I_{xx}}{P_{cg}A}$, $x_{cp} = -\gamma\sin\theta\frac{I_{xy}}{P_{cg}A}$



curved surface:



$F_H = P_{cg}A_{proj}$ on CP

$F_V = W_{above}$ on CG

F_H same

$F_V = W$ of missing water

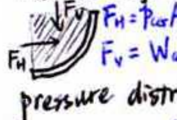
Buoyancy:

$B = W_{air} - W_w = \gamma_w V$ on CG

$W_{air} = \gamma_{thing} V = SG\,\gamma_w V$

$W_w = (SG - 1)\gamma_w V$

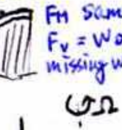
pressure distribution:



$\theta = \tan^{-1}\frac{a_x}{g + a_z}$

$\frac{dp}{ds} = \rho G$

$G = \sqrt{a_x^2 + (g + a_z)^2}$



$h = \frac{\Omega^2 R^2}{2g}$

$z = a + br^2$

$a = \frac{p_0 - p_1}{\gamma}$, $b = \frac{\Omega^2}{2g}$

$\Delta h = \Delta h_o + \Delta h_c$

$2\Delta h_o = \Delta h_c$

$\Delta h = \frac{\Omega^2 R^2}{2g}$

acceleration: $a = \frac{dV}{dt} = \frac{\partial V}{\partial t} + (V\cdot\nabla)V$ $\quad + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}$ steady flow: $\frac{\partial V}{\partial t} = 0$

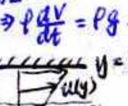
continuity: $\frac{\partial\rho}{\partial t} + \nabla\cdot\rho V = 0$ steady: $\frac{\partial\rho}{\partial t} = 0 \Rightarrow \frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w = 0$ incomp. $\rho = $ const. $\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

cylinderical: $\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$ steady; incomp: $\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$

momentum: $\frac{\partial\vec V}{\partial t} + (\vec V\cdot\nabla)\vec V = \rho g - \nabla p + \nabla\cdot\tau_{ij}$ incomp. $\rho\frac{dV}{dt} = \rho g - \nabla p + \mu\nabla^2 V$ inviscid $\mu = 0 \Rightarrow \rho\frac{dV}{dt} = \rho g - \nabla p$

Navier-stokes: $\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$

Couette flow: moving wall: $\frac{d^2u}{dy^2} = 0$, $u = C_1 y + C_2$ I.C. $\Rightarrow u(y) = \frac{V}{2h}y + \frac{V}{2}$, $-h \le y \le h$



two fixed plates: $\mu\frac{d^2u}{dy^2} = \frac{dp}{dx} = $ const. I.C. $\Rightarrow u = -\frac{h^2}{2\mu}\frac{dp}{dx}\left(1 - \frac{y^2}{h^2}\right)$

Fully developed: $\partial u/\partial t = 0$

cylinderical: r-mom: $\frac{\partial v_r}{\partial t} + (\vec V\cdot\nabla)v_r - \frac{1}{r}v_\theta^2 = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g_r + \nu\left(\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right)$

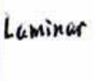
θ mom: $\frac{\partial v_\theta}{\partial t} + (V\cdot\nabla)v_\theta + \frac{1}{r}v_r v_\theta = -\frac{1}{\rho r}\frac{\partial p}{\partial\theta} + g_\theta + \nu\left(\nabla^2 v_\theta - \frac{v_\theta}{r^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right)$

z-mom: $\frac{\partial v_z}{\partial t} + (V\cdot\nabla)v_z = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z + \nu\nabla^2 v_z$

Laminar incomp. in pipe of R: $p = p(z)$, $\Rightarrow \rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$

$\Rightarrow v_z = \left(-\frac{dp}{dz}\right)\frac{1}{4\mu}(R^2 - r^2)$

concentric cylinders:



θ mom: $\rho(\nabla\cdot V)v_\theta + \frac{\rho v_\theta v_r}{r} = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \rho g_\theta + \mu\left(\nabla^2 v_\theta - \frac{v_\theta}{r^2}\right)$ $\quad \nabla^2 v_\theta = \frac{v_\theta}{r^2} \Rightarrow v_\theta = C_1 r + \frac{C_2}{r}$

I.C. $\Rightarrow v_\theta = \Omega_i r_i\left(\frac{r_o}{r} - \frac{r}{r_o}\right)/\left(\frac{r_o}{r_i} - \frac{r_i}{r_o}\right)$

Flow rate: $Q = VA$, $\dot m = \rho Q = \rho AV$ Re.T.T. $\frac{d}{dt}B_{syst} = \frac{d}{dt}\left(\int_{cv}\beta\rho\,d\forall\right) + \int\beta\rho\,\vec V\cdot\vec n\,dA$

one-D: $\frac{d}{dt}B_{syst} = \frac{d}{dt}\int_{cv}\beta\rho\,d\forall + \sum\beta_i\dot m_i|_{out} - \sum\beta_i\dot m_i|_{in}$, $\dot m_i = \rho_i A_i V_i$

Mass: for fixed $\forall$: $\sum\rho_i A_i V_i|_{in} = \sum\rho_i A_i V_i|_{out}$ avg vel. $V_{av} = \frac{Q}{A} = \frac{1}{A}\int\vec V\cdot\vec n\,dA$, $\rho_{av} = \frac{1}{A}\int\rho\,dA$

Momentum: $\sum\vec F = \frac{d}{dt}\left(\int_{cv}\vec V\rho\,d\forall\right) + \sum\dot m_i\vec V_i|_{out} - \sum\dot m_i\vec V_i|_{in}$, $\dot m_i = \rho_i A_i V_i$

pressure force: $\vec F_{press} = \int_{cs} P_{gage}(-\vec n)\,dA$ drag force: $D = \rho b\int_0^\delta u(U_0 - u)\,dy\big|_{x=L}$

correction factor: $\rho\int u^2 dA = \beta\dot m V_{av} = \beta\rho A V_{av}^2$, $\beta = \frac{4}{3}$ for laminar flow.

Ang. Momentum: $\sum\vec M_o = \frac{\partial}{\partial t}\left(\int_{cv}(\vec r\times\vec V)\rho\,d\forall\right) + \sum(\vec r\times\vec V)_{out}\dot m_{out} - \sum(\vec r\times\vec V)_{in}\dot m_{in}$

energy: +ve. $\begin{cases} W \text{ by syst.} \\ H \text{ to syst.}\end{cases}$ enthalpy: $h = u + P/\rho$, $e = h + \frac{1}{2}V^2 + gz$

steady flow: $\frac{dE}{dt} = \dot Q - \dot W_s - \dot W_v = \left(h + \frac{V^2}{2} + gz\right)_{out}\dot m_{out} - \left(h + \frac{V^2}{2} + gz\right)_{in}\dot m_{in}$

work: $\dot W = \dot W_s + \dot W_p + \dot W_v$, $\dot W_p = \int_{cs} p(\vec V\cdot\vec n)\,dA$, $\dot W_v = \int_{cs}\tau\cdot\vec V\,dA$ (often ignore)

head energy eqn: $\frac{p}{\gamma} + \frac{V^2}{2g} + z\Big|_{in} = \frac{p}{\gamma} + \frac{V^2}{2g} + z\Big|_{out} + h_f - h_p + h_t + h_m$

Bernoulli: $\int_1^2\frac{\partial V}{\partial t}ds + \int_1^2\frac{dp}{\rho} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0$

1-D, steady, incompressible: $\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = $ const

correction factor: $\alpha = 2.0$ for laminar, add to vel. head term, $\alpha = \frac{1}{A}\int\left(\frac{u}{V_{av}}\right)^3 dA$

Turbine/Pump power: $P = \gamma Q h_t$

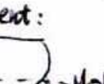
pipe flow: $Re_d = Vd/\nu = 4Q/\pi d\nu$ entrance region: $L_e/d = 0.06\,Re_d$ (Laminar) $L_e/d = 4.4\,Re_d^{1/6}$ (Turbulent)

Friction head: $h_f = f\frac{L}{d}\frac{V^2}{2g}$, $f = \frac{8\tau_w}{\rho V^2}$ Laminar: $\tau_w = \frac{4\mu V}{R}$, $f = \frac{64}{Re_d}$, $h_f = \frac{32\mu LV}{\rho g d^2} = \frac{128\mu LQ}{\pi\rho g d^4}$

mean value turb. modeling: $\rho\frac{d\bar u}{dt} = \rho g_x - \frac{\partial\bar p}{\partial x} + \frac{\partial}{\partial x}\left(\mu\frac{\partial\bar u}{\partial x} - \rho\overline{u'^2}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial\bar u}{\partial y} - \rho\overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial\bar u}{\partial z} - \rho\overline{u'w'}\right)$

where $\bar u = \frac{1}{T}\int_0^T u\,dt$ and $u' = u - \bar u$

Turbulent:



velocity profile: $\frac{u(r)}{u^*} = \frac{1}{k}\ln\frac{(R - r)u^*}{\nu} + B$, $u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2}$, $k = 0.4$, $B = 5.0$

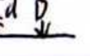
$f = \begin{cases} 0.316\,Re_d^{-1/4} & 4000 < Re_d < 10^5 \\ \left(1.8\log\frac{Re_d}{6.9}\right)^{-2} \end{cases}$ horizontal pipe: $\Delta p = 0.158\,L\rho^{3/4}\mu^{1/4}d^{-5/4}V^{7/4}$
$= 0.24\,L\rho^{3/4}\mu^{1/4}d^{-4.75}Q^{1.75}$

rough wall: $u^+ = \frac{u}{u^*} = \frac{1}{k}\ln\left(\frac{y}{\varepsilon}\right) + 8.5$, $\frac{1}{f^{1/2}} = -2\log\left(\frac{\varepsilon/d}{3.7}\right)$

Smooth & rough wall: $\frac{1}{\sqrt f} = 1.8\log\left[\frac{6.9}{Re_d} + \left(\frac{\varepsilon/d}{3.7}\right)^{1.11}\right]$ iteration: have $f = f_{cn}(V)$, guess $f = 0.02$
by find $V \Rightarrow Re_d \Rightarrow f_{new}$, stop if $f_{new} = f$

Hydraulic diameter for non-circular pipe: $D_h = \frac{4A}{P}$ sharp entrance $K = 0.5$
submerged exit $K = 1.0$

Minor loss: $K = \frac{h_m}{V^2/2g} = \frac{\Delta p}{\rho V^2/2}$, $\Delta h_{tot} = h_f + \sum h_m = \frac{V^2}{2g}\left(f\frac{L}{d} + \sum K\right)$ $K_{SE} = \left(1 - \frac{d^2}{D^2}\right)^2$ $K_{SC} = 0.42\left(1 - \frac{d^2}{D^2}\right)$



$Q = C_d A_t\left[\frac{2(p_1 - p_2)/\rho}{1 - \beta^4}\right]^{1/2}$, $A_t = \frac{\pi}{4}d^2$, $\beta = d/D$, C_d relates to Re, read from plot

Scaling Laws: e.g. $Re_m = Re_p$ $\quad \frac{F_p}{F_m} = \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2$

Π-theorem: ① list n variables and their dimensions

② find j (typically 3), select j parameters that can't form a Π it self $(a = b = c = 0)$

③ write $\Pi_1 = P_1^a P_2^b P_3^c V = M^0 L^0 T^0$, solve for a, b, c ($P_1 \sim 1^{st}$ parameter
$P_2 \sim 2^{nd}$ "
$P_3 \sim 3^{rd}$ "
$V \sim$ interested variable)

④ $\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots \Pi_k)$, where $k = n - j$